39

$$\boxed{\int \frac{1}{3x + \sqrt[3]{x^2}}\,dx}$$

$$= \int \frac{1}{3x + (\sqrt[3]{x})^2}\,dx \qquad x = t^3,\ dx = 3t^2\,dt$$

$$= \int \frac{1}{3t^3 + (\sqrt[3]{t^3})^2}\,3t^2\,dt = 3\int \frac{1}{3t^3 + t^2}\,t^2\,dt$$

$$= 3\int \frac{1}{t^2(3t+1)}\,t^2\,dt = 3\int \frac{1}{3t+1}\,dt \qquad 3t+1 = k,\ 3dt = dk,\ dt = \frac{dk}{3}$$

$$= 3\int \frac{1}{k}\,\frac{dk}{3} = \frac{3}{3}\int \frac{1}{k}\,dk$$

$$= \frac{3}{3}\ln k = \frac{3}{3}\ln|3t+1| = \boxed{\ln|\sqrt[3]{x}+1| + C}$$

40

$$\boxed{\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})}\,dx}$$

$$\int \frac{x + (\sqrt[3]{x})^2 + \sqrt[6]{x}}{x(1+\sqrt[3]{x})}\,dx \Rightarrow \qquad x = t^6,\ dx = 6t^5\,dt$$

$$\int \frac{t^6 + (\sqrt[3]{t^6})^2 + \sqrt[6]{t^6}}{t^6(1+\sqrt[3]{t^6})}\cdot 6t^5\,dt$$

$$6\int \frac{t^6 + t^4 + t}{t(1+t^2)}\,dt = 6\int \frac{t(t^5+t^3+1)}{t(1+t^2)}\,dt$$

$$6\int \frac{t^5+t^3+1}{t^2+1}\,dt$$

$$t^5 + t^3 + 1 : t^2+1 = t^3 + \frac{1}{t^2+1}$$

$$\underline{-t^5 \pm t^3}$$

$$1$$

$$= 6\left(\int t^3\,dt + \int \frac{1}{t^2+1}\,dt\right) = 6\left(\frac{t^4}{4} + \operatorname{arctg} t\right)$$

$$\boxed{= 6\cdot\left(\frac{(\sqrt[6]{x})^4}{4} + \operatorname{arctg}\sqrt[6]{x}\right) + C}$$

41

$$\boxed{\int \frac{\sqrt{2x-3}}{\sqrt[3]{2x-3}+1}\,dx}$$

$$2x-3 = t^6,\quad 2dx = 6t^5\,dt,\quad dx = 3t^5\,dt$$

$$\int \frac{\sqrt{t^6}}{\sqrt[3]{t^6}+1}\,3t^5\,dt$$

$$3\int \frac{t^3}{t^2+1}\,t^5\,dt = 3\int \frac{t^8}{t^2+1}\,dt$$

$$t^8 : t^2+1 = t^6 - t^4 + t^2 - 1 + \frac{1}{t^2+1}$$

$$\underline{-t^8 \pm t^6}$$

$$-t^6$$

$$\mp t^6 \mp t^4$$

? pitati Papu

$$3\left(\int t^6\,dt - \int t^4\,dt + \int t^2\,dt - \int dt + \int \frac{1}{t^2+1}\,dt\right)$$

$$3\left(\frac{t^7}{7} - \frac{t^5}{5} + \frac{t^3}{3} - t + \operatorname{arctg} t\right)$$

$$3\left(\frac{(\sqrt[6]{2x-3})^7}{7} - \frac{(\sqrt[6]{2x-3})^5}{5} + \frac{(\sqrt[6]{2x-3})^3}{3} - \sqrt[6]{2x-3} + \operatorname{arctg}\sqrt[6]{2x-3}\right)$$

42

$$\boxed{\int \frac{\sqrt{x+1}}{x}\,dx} \qquad x+1 = t^2 \Rightarrow x = t^2-1,\quad dx = 2t\,dt$$

$$\int \frac{\sqrt{t^2}}{t^2-1}\,2t\,dt = 2\int \frac{t^2}{t^2-1}\,dt$$

$$t^2 : t^2-1 = 1 + \frac{1}{t^2-1} \qquad = 2\int 1 + \frac{1}{t^2-1}\,dt$$

$$\underline{-t^2 \mp 1}$$

$$1$$

$$= 2\left[t + \frac{1}{2}\ln\frac{t-1}{t+1}\right] = 2t + \ln\frac{t-1}{t+1}$$

$$\boxed{= 2\sqrt{x+1} + \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} + C}$$

43

$$\boxed{\int \frac{1}{x-1}\sqrt{\frac{x+1}{x-1}}\,dx} \qquad -\frac{2}{(x-1)^2}\,dx$$

$$\frac{x+1}{x-1} = t^2$$

$$x+1 = xt^2 - t^2$$

$$x - xt^2 = -t^2 - 1$$

$$x(1-t^2) = -(t^2+1)$$

$$x = -\frac{t^2+1}{1-t^2}$$

$$x = \frac{t^2+1}{t^2-1}$$

$$dx = \frac{2t(t^2-1) - (t^2+1)2t}{(t^2-1)^2}\,dt$$

$$dx = \frac{2t(\cancel{t^2}-1-\cancel{t^2}-1)}{(t^2-1)^2}\,dt$$

$$dx = -\frac{4t}{(t^2-1)^2}\,dt$$

$$\int \frac{1}{\frac{t^2+1}{t^2-1} - 1}\cdot\sqrt{t^2}\,\frac{-4t}{(t^2-1)^2}\,dt$$

$$-4\int \frac{1}{\frac{t^2+1-t^2+1}{t^2-1}}\cdot\frac{t^2}{(t^2-1)^2}\,dt = -4\int \frac{1}{2}\,\frac{t^2}{(t^2-1)}\,dt$$

$$-2\int \frac{t^2}{t^2-1}\,dt \qquad t^2 : t^2-1 = 1 + \frac{1}{t^2-1}$$

$$\underline{-t^2 \mp 1}$$

$$1$$

$$-2\int\left(1 + \frac{1}{t^2-1}\right)dt = -2\left(t + \frac{1}{2}\ln\frac{t-1}{t+1}\right)$$

$$\boxed{= -2\sqrt{\frac{x+1}{x-1}} - \ln\frac{\sqrt{\frac{x+1}{x-1}}-1}{\sqrt{\frac{x+1}{x-1}}+1} + C}$$

(44) $\boxed{\int \frac{4}{(x-1)^2} \sqrt[3]{\frac{1-x}{1+x}}\, dx}$

$\frac{1-x}{1+x} = t^3$

$1-x = xt^3 + t^3$

$-x - xt^3 = t^3 - 1$

$-x(1+t^3) = t^3 - 1$

$x(1+t^3) = 1 - t^3$

$x = \frac{1-t^3}{1+t^3}$

$dx = \frac{-3t^2(1+t^3) - (1-t^3)3t^2}{(1+t^3)^2} dt$

$dx = \frac{3t^2(-1-\cancel{t^3}-1+\cancel{t^3})}{(1+t^3)^2} dt$

$dx = \frac{-6t^2}{(1+t^3)^2} dt$

$4 \cdot \int \frac{1}{\left(\frac{1-t^3}{1+t^3} - 1\right)^2} \sqrt[3]{t^3} \frac{-6t^2}{(1+t^3)^2} dt$

$-24 \int \frac{1}{\frac{(1-t^3-1-t^3)^2}{\cancel{(t^3+1)^2}}} \frac{t^3}{\cancel{(t^3+1)^2}} dt$

$-24 \int \frac{1}{4t^6} t^3 dt = -\frac{24}{4} \int \frac{1}{t^3} dt$

$= -6\left(-\frac{1}{2t^2}\right) = \frac{3}{\left(\sqrt[3]{\frac{1-x}{1+x}}\right)^2} = \boxed{3\left(\sqrt[3]{\frac{1+x}{1-x}}\right)^2 + C}$

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(45) $\boxed{\int \frac{1}{x\sqrt{x^2-9}}\, dx}$

što ta smjena ?

$x = \frac{1}{t}$

$dx = -\frac{1}{t^2} dt$

$\int \frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}-9}} \left(-\frac{1}{t^2}\right) dt$

$-\int \frac{1}{\frac{1}{\cancel{t}}\sqrt{\frac{1-9t^2}{t^2}}} \frac{1}{\cancel{t^2}} dt = -\int \frac{1}{\frac{\sqrt{1-9t^2}}{\cancel{\sqrt{t^2}}}} \frac{dt}{\cancel{t}}$

$-\int \frac{(1)}{\sqrt{1-(3t)^2}} dt =$

$3t = k$

$3dt = dk$

$dt = \frac{dk}{3}$

$-\int \frac{1}{\sqrt{1-k^2}} \frac{dk}{3}$

$-\frac{1}{3} \int \frac{1}{\sqrt{1-k^2}} dk = -\frac{1}{3} \arcsin k$

nije točno ili ?

$= -\frac{1}{3} \arcsin 3t$ $\boxed{= -\frac{1}{3} \arcsin \frac{3}{x} + C}$

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(46) $\boxed{\int \frac{1}{x^2\sqrt{x^2+4}}\, dx}$

$x = \frac{1}{t}$

$dx = -\frac{1}{t^2} dt$

$-\int \frac{1}{\frac{1}{t^2}\sqrt{\frac{1}{t^2}+4}} \frac{dt}{\cancel{t^2}} = -\int \frac{1}{\sqrt{\frac{1+4t^2}{t^2}}} dt$

$-\int \frac{1}{\frac{\sqrt{1+4t^2}}{\sqrt{t^2}}} dt = -\int \frac{t}{\sqrt{1+4t^2}} dt$

$-\int \frac{t}{\sqrt{1+4t^2}} dt$

$1+4t^2 = k^2$

$8t\, dt = 2k\, dk$

$dt = \frac{k}{4t} dk$

$-\int \frac{t}{\sqrt{k^2}} \frac{k}{4t} dk$

$-\frac{1}{4}\int dk = -\frac{1}{4} k = -\frac{1}{4}\sqrt{1+4t^2}$

$\boxed{= -\frac{1}{4}\sqrt{1+\frac{4}{x^2}} + C}$

(96)

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(48) $\boxed{\int \frac{1}{(x+1)\sqrt{x^2+2x-3}}\, dx}$ što

što ta smjena

$x+1 = \frac{1}{t} \Rightarrow x = \frac{1}{t}$

$dx = -\frac{1}{t^2} dt$

$-\int \frac{1}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2 + 2\left(\frac{1}{t}-1\right) - 3}} \frac{1}{t^2} dt$

$-\int \frac{1}{\sqrt{\frac{(1-t)^2}{t^2} + \frac{2}{t} - 2 - 3}} \frac{dt}{t}$

$-\int \frac{1}{\sqrt{\frac{1-2t+t^2+2t-5t^2}{t^2}}} \frac{dt}{t} = -\int \frac{1}{\frac{\sqrt{-4t^2+1}}{\sqrt{t^2}}} \frac{dt}{t}$

$-\int \frac{1}{\sqrt{1-(2t)^2}} dt$

$2t = k$

$2dt = dk$

$dt = \frac{dk}{2}$

$-\int \frac{1}{\sqrt{1-k^2}} \frac{dk}{2}$

$-\frac{1}{2} \arcsin k = -\frac{1}{2} \arcsin 2t$

$= -\frac{1}{2} \arcsin 2 \cdot \left(\frac{1}{x+1}\right)$

$\boxed{= -\frac{1}{2} \arcsin \frac{2}{x+1} + C}$

?

$t = \frac{1}{x}$

$\frac{\frac{1}{1}}{\frac{1}{x^2}}$