$$\int u\,dv = uv - \int v\,du$$

Integrali – parcijalna integracija

57) $\int (x^2-1)e^x dx$ $u = x^2-1$, $dv = e^x dx$, $du = 2x\,dx$, $v = \int e^x dx$, $v = e^x$

$= (x^2-1)e^x - \int e^x 2x\,dx$

$= (x^2-1)e^x - 2\int x e^x dx$ $u = x$, $dv = e^x dx$, $du = dx$, $v = \int e^x dx$, $v = e^x$

$= (x^2-1)e^x - 2(xe^x - \int e^x dx)$

$= (x^2-1)e^x - 2xe^x + 2e^x$

$= e^x(x^2-1-2x+2) = e^x(x^2-2x+1)$

$= \boxed{e^x(x-1)^2 + C}$

58) $\int \frac{x}{\sin^2 x} dx$ $u = x$, $dv = \frac{1}{\sin^2 x}dx$, $du = dx$, $v = \int \frac{1}{\sin^2 x} dx$, $v = -\mathrm{ctg}\,x$

$I = -x\,\mathrm{ctg}\,x + \int \mathrm{ctg}\,x\,dx$

$I = -x\,\mathrm{ctg}\,x + \int \frac{\cos x}{\sin x} dx$ $\sin x = t$, $\cos x\,dx = dt$, $dx = \frac{dt}{\cos x}$

$I = -x\,\mathrm{ctg}\,x + \int \frac{\cos x}{t}\frac{dt}{\cos x}$

$I = -x\,\mathrm{ctg}\,x + \ln t$

$\boxed{I = -x\,\mathrm{ctg}\,x + \ln|\sin x| + C}$

59) $\int x^2 \sin x\,dx$

$u = x^2$, $dv = \sin x\,dx$, $du = 2x\,dx$, $v = \int \sin x\,dx$, $v = -\cos x$

$I = -x^2\cos x + 2\int x\cos x\,dx$

$u = x$, $dv = \cos x\,dx$, $du = dx$, $v = \int \cos x\,dx$, $v = +\sin x$

$I = -x^2\cos x + 2(+x\sin x - \int \sin x\,dx)$

$\boxed{I = -x^2\cos x + 2x\sin x + 2\cos x + C}$

60) $\int x e^{-x} dx$ $u = x$, $dv = e^{-x}dx$, $du = dx$, $v = \int e^{-x}dx$, $v = -e^{-x}$

$I = -xe^{-x} + \int e^{-x}dx$

$I = -xe^{-x} - e^{-x}$

$\boxed{I = -e^{-x}(x+1) + C}$

$\int x e^{2x} dx$ $u = x$, $dv = e^{2x}dx$, $du = dx$, $v = \int e^{2x}dx$; $2x = t$, $2dx = dt$, $dx = \frac{dt}{2}$, $v = \int e^t \frac{dt}{2}$, $v = \frac{1}{2}e^{2x}$

$I = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$

$I = \frac{1}{2}xe^{2x} - \frac{1}{2}\cdot\frac{1}{2}e^{2x}$

$\boxed{I = \frac{1}{2}e^{2x}(x - \frac{1}{2}) + C}$

61) $\int (x^2-3x+2)e^x dx$ $u = (x^2-3x+2)$, $dv = e^x dx$, $du = (2x-3)dx$, $v = \int e^x dx$, $v = e^x$

$I = (x^2-3x+2)e^x - \int e^x(2x-3)dx$ $u = 2x-3$, $dv = e^x dx$, $du = 2dx$, $v = e^x$

$I = (x^2-3x+2)e^x - [(2x-3)e^x - \int e^x \cdot 2dx]$

$I = (x^2-3x+2)e^x - (2x-3)e^x + 2e^x$

$I = e^x(x^2-3x+3-2x+3+2)$

$\boxed{I = e^x(x^2-5x+7) + C}$

62) $\int x\cdot\ln x\,dx$ $u = \ln x$, $dv = x\,dx$, $du = \frac{1}{x}dx$, $v = \int x\,dx$, $v = \frac{x^2}{2}$

$I = \frac{x^2}{2}\ln x - \int \frac{1}{x}\frac{x^2}{2}dx$

$I = \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2} = \frac{x^2}{2}(\ln x - \frac{1}{2}) + C$

63) $\int \ln x\,dx$ $u = \ln x$, $dv = dx$, $du = \frac{1}{x}dx$, $v = \int dx$, $v = x$

$I = x\ln x - \int x\frac{1}{x}dx$

$= x\ln x - x \Rightarrow \boxed{x(\ln x - 1) + C}$

64) $\int x^2 \mathrm{arctg}\,x\,dx$ $u = \mathrm{arctg}\,x$, $dv = x^2dx$, $du = \frac{1}{x^2+1}dx$, $v = \int x^2 dx$, $v = \frac{x^3}{3}$

$I = \frac{x^3}{3}\mathrm{arctg}\,x - \frac{1}{3}\int \frac{x^3}{x^2+1}dx$ $x^3 : (x^2+1) = x - \frac{x}{x^2+1}$, $\frac{-x^3 \mp x}{-x}$

$I = \frac{x^3}{3}\mathrm{arctg}\,x - \frac{1}{3}\int(x - \frac{x}{x^2+1})dx$

$I = \frac{x^3}{3}\mathrm{arctg}\,x - \frac{1}{3}[\frac{x^2}{2} - \int\frac{x}{x^2+1}dx]$ $x^2+1 = t$, $2x\,dx = dt$, $dx = \frac{dt}{2x}$

$I = \frac{x^3}{3}\mathrm{arctg}\,x - \frac{1}{3}(\frac{x^2}{2} - \int\frac{x}{t}\frac{dt}{2x}]$

$\boxed{I = \frac{x^3}{3}\mathrm{arctg}\,x - \frac{1}{3}(\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1|) + C}$

65) $\int \arcsin x\,dx$ $u = \arcsin x$, $dv = dx$, $du = \frac{1}{\sqrt{1-x^2}}dx$, $v = \int dx$, $v = x$

$I = x\arcsin x - \int\frac{x}{\sqrt{1-x^2}}dx$ $1-x^2 = t^2$, $-2x\,dx = 2t\,dt$, $dx = -\frac{t}{x}dt$

$I = x\arcsin x + \int\frac{x}{\sqrt{t^2}}\frac{t}{x}dt$

$I = x\arcsin x + t$

$\boxed{I = x\arcsin x + \sqrt{1-x^2} + C}$

(66) $\boxed{\int \ln(x^2+1)\,dx}$ $u=\ln(x^2+1)$ $dv=dx$, $du=\frac{2x}{x^2+1}dx$ $V=\int dx$, $V=x$

$= x\ln(x^2+1) - 2\int \frac{x^2}{x^2+1}dx$

$= x\ln(x^2+1) - 2\int 1-\frac{1}{x^2+1}dx$

$\boxed{= x\ln|x^2+1| - 2x + 2\,\mathrm{arctg}\,x + C}$

(67) $\boxed{\int \frac{\ln x}{x^2}dx}$ $u=\ln x$ $dv=\frac{1}{x^2}dx$, $du=\frac{1}{x}dx$ $V=\int\frac{1}{x^2}dx$, $V=-\frac{1}{x}$

$= -\frac{1}{x}\ln x + \int \frac{1}{x^2}dx$

$= -\frac{1}{x}\ln x - \frac{1}{x} = \boxed{-\frac{1}{x}(\ln x+1)+C}$

(68) $\boxed{\int e^{2x}\cos 3x\,dx}$ $u=e^{2x}$ $dv=\cos 3x\,dx$, $du=2e^{2x}$ $V=\int\cos 3x\,dx$, $V=\frac{1}{3}\sin 3x$

?

$I=\frac{1}{3}e^{2x}\sin 3x-\frac{2}{3}\int e^{2x}\sin 3x\,dx$

$u=e^{2x}$ $dv=\sin 3x\,dx$, $du=2e^{2x}dx$ $V=\int\sin 3x\,dx$, $V=-\frac{1}{3}\cos 3x$

$I=\frac{1}{3}e^{2x}\sin 3x-\frac{2}{3}\left(-\frac{1}{3}e^{2x}\cos 3x+\frac{2}{3}\int e^{2x}\cos 3x\,dx\right)$

$I=\frac{1}{3}e^{2x}\sin 3x+\frac{2}{3}\frac{1}{3}e^{2x}\cos 3x-\frac{4}{9}I$

$I+\frac{4}{9}I=\frac{1}{3}e^{2x}\left(\sin 3x+\frac{2}{3}\cos 3x\right)$

$\frac{13}{9}I=\frac{1}{3}e^{2x}\left(\sin 3x+\frac{2}{3}\cos 3x\right)$

$\boxed{I=\frac{3}{13}e^{2x}\left(\sin 3x+\frac{2}{3}\cos 3x\right)+C}$

(69) $\boxed{\int e^{3x}\sin 2x\,dx}$ $u=e^{3x}$ $dv=\sin 2x\,dx$, $du=3e^{3x}dx$ $V=\int\sin 2x\,dx$, $V=-\frac{1}{2}\cos 2x$

$I=-\frac{1}{2}e^{3x}\cos 2x+\frac{3}{2}\int e^{3x}\cos 2x\,dx$

$u=e^{3x}$ $dv=\cos 2x$, $du=3e^{3x}dx$ $V=\int\cos 2x\,dx$, $V=\frac{1}{2}\sin 2x$

$I=-\frac{1}{2}e^{3x}\cos 2x+\frac{3}{2}\left(\frac{1}{2}e^{3x}\sin 2x-\frac{3}{2}\int e^{3x}\sin 2x\,dx\right)$

$I=-\frac{1}{2}e^{3x}\cos 2x+\frac{3}{2}\frac{1}{2}e^{3x}\sin 2x-\frac{9}{4}I$

$I+\frac{9}{4}I=\frac{1}{2}e^{3x}\left(-\cos 2x+\frac{3}{2}\sin 2x\right)$

$\frac{13}{4}I=\frac{1}{2}e^{3x}\left(-\cos 2x+\frac{3}{2}\sin 2x\right)$

$\boxed{I=\frac{2}{13}e^{3x}\left(-\cos 2x+\frac{3}{2}\sin 2x\right)+C}$

(70) $\boxed{\int \sin(\ln x)\,dx}$ $\ln x=t \Rightarrow e^{\ln x}=e^t$, $\frac{1}{x}dx=dt$ $x=e^t$, $dx=x\,dt$

$\int x\sin t\,dt$

$\int e^t\sin t\,dt$

$u=e^t$ $du=e^t dt$ $v=\int\sin t\,dt$, $V=-\cos t$

$I=-e^t\cos t+\int e^t\cos t\,dt$

$u=e^t$ $dv=\cos t\,dt$, $du=e^t dt$ $V=\int\cos t\,dt$, $V=\sin t$

$I=-e^t\cos t+\left(e^t\sin t-\int e^t\sin t\,dt\right)$

$I=-e^t\cos t+e^t\sin t-I$

$2I=e^t(-\cos t+\sin t)$

$I=\frac{1}{2}e^{\ln x}\left(\sin(\ln x)-\cos(\ln x)\right)$

$\boxed{I=\frac{x}{2}\left(\sin(\ln x)-\cos\ln x\right)+C}$

(71) $\boxed{\int\sqrt{x^2+4}\,dx}$ $\cdot\frac{\sqrt{x^2+4}}{\sqrt{x^2+4}}$

$=\int\frac{x^2+4}{\sqrt{x^2+4}}dx=\int x\frac{x}{\sqrt{x^2+4}}dx+4\int\frac{1}{\sqrt{x^2+4}}dx$

$u=x$ $dv=\frac{x}{\sqrt{x^2+4}}dx$, $du=dx$ $V=\int\frac{x}{\sqrt{t}}\frac{dt}{x}$, $V=t=\sqrt{x^2+4}$

$I=x\sqrt{x^2+4}-\int\sqrt{x^2+4}\,dx+4\ln|x+\sqrt{x^2+4}|$

$I=x\sqrt{x^2+4}-I+4\ln|x+\sqrt{x^2+4}|$

$2I=x\sqrt{x^2+4}+4\ln|x+\sqrt{x^2+4}|$

$\boxed{I=\frac{x}{2}\sqrt{x^2+4}+2\ln|x+\sqrt{x^2+4}|+C}$

(72) $\boxed{\int\frac{x^2}{\sqrt{9-x^2}}dx}$ $u=x$ $dv=\frac{x}{\sqrt{9-x^2}}dx$, $du=dx$ $V=\int\frac{x}{\sqrt{9-x^2}}dx$

$\int x\cdot\frac{x}{\sqrt{9-x^2}}dx$ $9-x^2=t$, $dx=-\frac{1}{x}dt$ $V=-\int\frac{x}{\sqrt{t}}\frac{1}{x}dt$, $V=-t=-\sqrt{9-x^2}$

$I=-x\sqrt{9-x^2}+\int\sqrt{9-x^2}\,dx\cdot\frac{\sqrt{9-x^2}}{\sqrt{9-x^2}}$

$I=-x\sqrt{9-x^2}+\int\frac{9-x^2}{\sqrt{9-x^2}}dx$???

$I=-x\sqrt{9-x^2}+9\int\frac{1}{\sqrt{9-x^2}}-\int\frac{x^2}{\sqrt{9-x^2}}dx$

$I=-x\sqrt{9-x^2}+9\cdot\arcsin\frac{x}{3}-I$ zašto tablični

$2I=-x\sqrt{9-x^2}+9\arcsin\frac{x}{3}$

$\boxed{I=-\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\arcsin\frac{x}{3}+C}$

(73) $\boxed{\int\sqrt{1-x^2}\,dx}$ $\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$ $1-x^2=t$, $dx=-\frac{1}{x}dt$ $u=x$ $dv=\frac{x}{\sqrt{1-x^2}}dx$, $du=dx$ $V=-\int\frac{x}{\sqrt{t}}\frac{1}{x}dt$, $V=-t$, $V=-\sqrt{1-x^2}$

$\int\frac{1-x^2}{\sqrt{1-x^2}}dx=\int\frac{1}{\sqrt{1-x^2}}dx-\int x\frac{x}{\sqrt{1-x^2}}dx$

$I=\arcsin x-\left(-x\sqrt{1-x^2}+\int\sqrt{1-x^2}\,dx\right)$

$I=\arcsin x+x\sqrt{1-x^2}-I$

$2I=\arcsin x+x\sqrt{1-x^2}$

$\boxed{I=\frac{1}{2}\arcsin x+\frac{x}{2}\sqrt{1-x^2}+C}$

?

Tablični?

11. HE: 75, 78, 79, 80, 82, 85, 86, 88, 91–96, 99, 100, 103–115, 118–90kp

↳ ZA SADA :)

Integrali – Koktel

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(74) $\boxed{\int \frac{1}{1+e^{-x}}\,dx}$

$\int \frac{1}{1+\frac{1}{e^x}}\,dx = \int \frac{e^x}{e^x+1}\,dx$ $\qquad e^x+1=t,\ e^x dx = dt,\ dx=\frac{dt}{e^x}$

$\int \frac{e^x}{t}\,\frac{dt}{e^x} = \int \frac{1}{t}\,dt = \ln t = \boxed{\ln|e^x+1|+C}$

*!

(75) $\boxed{\int \operatorname{arctg}\sqrt{x}\,dx}$

2ABS(rAvljena kasnije :)

$x=t^2$, $dx = 2t\,dt$

ne razumijem

$= \int \operatorname{arctg}\sqrt{t^2}\;2t\,dt = (2)\int t \operatorname{arctg} t\,dt$

$u = \operatorname{arctg} t \quad dv = t\,dt$

$du = \frac{1}{t^2+1}dt \quad v = \int t\,dt,\ v=\frac{t^2}{2}$

$2\cdot = \frac{t^2}{2}\operatorname{arctg} t - \int \frac{t^2}{2}\frac{1}{t^2+1}dt$

$2\cdot = \frac{t^2}{2}\operatorname{arctg} t - \frac{1}{2}\int \frac{t^2}{t^2+1}dt$

$2\cdot = \frac{t^2}{2}\operatorname{arctg} t - \frac{1}{2}\int 1-\frac{1}{t^2+1}\,dt$

$2\cdot = \frac{t^2}{2}\operatorname{arctg} t - \frac{1}{2}(t - \operatorname{arctg} t)$

$\boxed{= 2\cdot\frac{x}{2}\operatorname{arctg}\sqrt{x} - 2\cdot\frac{\sqrt{x}}{2} + 2\cdot\frac{1}{2}\operatorname{arctg}\sqrt{x} + C}$

*

(76) $\boxed{\int \frac{-x^2-4}{x^3+3x^2-4x}\,dx}$ $\qquad x_{1,2} = \frac{-3\pm\sqrt{9+16}}{2} \lessgtr {1 \atop -4}$

$\frac{-x^2-4}{x(x^2+3x-4)} = \frac{-x^2-4}{x(x-1)(x+4)}$

$\frac{-x^2-4}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4} \quad \Big/\ x(x-1)(x+4)$

x^2+3x-4

$-x^2-4 = A(x-1)(x+4) + Bx(x+4) + Cx(x-1)$

$-x^2-4 = x^2(A+B+C) + x(3A+4B-C) + (-4A)$

$A+B+C = -1$
$3A+4B-C = 0$
$-4A = -4 \Rightarrow \boxed{A=1}$

$B+C=-2$
$4B-C=-3$ $\Rightarrow 5B = 5$

$\boxed{B=-1} \quad \boxed{C=-1}$

$\int \frac{1}{x}dx + \int \frac{-1}{x-1}dx + \int \frac{-1}{x+4}dx$

$\boxed{\ln x - \ln|x-1| - \ln|x+4| + C}$

*

(77) $\boxed{\int \left(\frac{\ln x+1}{\ln x-1}\right)^2 \frac{dx}{x\ln x}}$ $\qquad \ln x = t,\ \frac{1}{x}dx = dt,\ dx = x\,dt$

$\int \left(\frac{t+1}{t-1}\right)^2 \frac{1}{\cancel{x}t}\cancel{x}\,dt$

$\int \frac{t^2+2t+1}{(t-1)^2 t}\,dt$

$\frac{t^2+2t+1}{t(t-1)^2} = \frac{A}{t} + \frac{B}{(t-1)^2} + \frac{C}{t-1} \quad \Big/\ t(t-1)^2$

$t^2+2t+1 = A(t^2-2t+1) + Bt + Ct(t-1)$

$t^2+2t+1 = t^2(A+C) + t(-2A+B-C) + (A)$

$A+C=1 \Rightarrow \boxed{C=0}$
$-2A+B-C=2 \Rightarrow \boxed{B=4}$
$A = 1 \Rightarrow \boxed{A=1}$

$\int \frac{1}{t}dt + \int \frac{4}{(t-1)^2}dt$ $\qquad t-1=k,\ dt=dk$

$\ln t + 4\int \frac{1}{k^2}dk = \ln t - \frac{4}{k}$

$\ln t - \frac{4}{t-1} = \boxed{\ln|\ln x| - \frac{4}{\ln x - 1} + C}$

*

$\boxed{\int \frac{e^{2x}+e^x}{(e^x+2)(e^{2x}+1)}\,dx}$ $\qquad e^x = t,\ e^x dx = dt,\ dx = \frac{dt}{e^x} = \frac{dt}{t}$

$\int \frac{t^2+t}{(t+2)(t^2+1)}\,\frac{dt}{t}$

$\int \frac{\cancel{t}(t+1)}{(t+2)(t^2+1)}\frac{dt}{\cancel{t}} = \int \frac{t+1}{(t+2)(t^2+1)}\,dt$

$\frac{t+1}{(t+2)(t^2+1)} = \frac{A}{t+2} + \frac{Bt+C}{t^2+1} \quad \Big/ (t+2)(t^2+1)$

$t+1 = A(t^2+1) + (Bt+C)(t+2)$

$t+1 = At^2 + A + Bt^2 + 2Bt + Ct + 2C$

$t+1 = t^2(A+B) + t(2B+C) + (A+2C)$

$A+B=0$
$2B+C=1$
$A+2C=1$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 1 \end{pmatrix} \Big/ \cdot(-1)$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & -1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 2 & 1 & 1 \end{pmatrix} \Big/ \cdot 2$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 5 & 3 \end{pmatrix} \Rightarrow$ $\quad A+B=0 \quad \boxed{A=-\frac{1}{5}}$

$-B+2C=1 \quad \boxed{B=\frac{1}{5}}$

$5C=3 \Rightarrow \boxed{C=\frac{3}{5}}$

$\int \frac{-\frac{1}{5}}{t+2}dt + \int \frac{\frac{1}{5}t+\frac{3}{5}}{t^2+1}\,dt$

$-\frac{1}{5}\ln|t+2| + \frac{1}{5}\int \frac{t+3}{t^2+1}\,dt$ $\qquad A=1,\ B=3 \quad a=1,\ b=0,\ c=1$

$-\frac{1}{5}\ln|t+2| + \frac{1}{5}\left(\frac{1}{2}\ln|t^2+1| + \left(3-\frac{0}{2}\right)\int\frac{1}{t^2+1}dt\right)$

$-\frac{1}{5}\ln|t+2| + \frac{1}{10}\ln|t^2+1| + \frac{3}{5}\operatorname{arctg} t$

$\boxed{-\frac{1}{5}\ln|e^x+2| + \frac{1}{10}\ln|e^{2x}+1| + \frac{3}{5}\operatorname{arctg} e^x + C}$

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(78) $\boxed{\int \frac{e^{tg x}\cdot \sin x}{\cos^3 x}\,dx}$

$tg x = t$

$\frac{1}{\cos^2 x}dx = dt$

$dx = \cos^2 x\, dt$

$\int \frac{e^t \cdot \sin x}{\cos^{\cancel{3}} x}\cdot \cancel{\cos^2 x}\,dt$

$\int \frac{e^t \sin x}{\cos x}\,dt = \int e^t \cdot tg x\, dt$

$\int t e^t dt \qquad u = t \quad dv = e^t dt$
$\qquad du = dt \quad v = e^t$

$te^t - \int e^t dt$

$te^t - e^t = e^t(t-1) = \boxed{e^{tg x}(tg x - 1) + C}$

---

(79) $\boxed{\int \frac{1}{\sin^2\sqrt{x}}\,dx}$

$x = t^2$
$dx = 2t\,dt$

$2\int \frac{t}{\sin^2 t}\,dt \qquad u = t \quad dv = \frac{1}{\sin^2 t}dt$
$\qquad du = dt \quad v = \int \frac{1}{\sin^2 t}dt$
$\qquad v = -ctg\, t$

$2\cdot\left(-t\,ctg\,t + \int ctg\,t\,dt\right)$

$-2t\,ctg\,t + 2\int \frac{\cos t}{\sin t}dt \qquad \sin t = k$
$\qquad \cos t\,dt = dk$

$-2t\cdot ctg\,t + 2\int \frac{\cancel{\cos t}}{k}\frac{dk}{\cancel{\cos t}} \qquad dt = \frac{dk}{\cos t}$

$-2t\,ctg\,t + 2\ln k$

$-2t\,ctg\,t + 2\ln|\sin t|$

$\boxed{-2\sqrt{x}\,ctg\sqrt{x} + 2\ln|\sin\sqrt{x}| + C}$

---

(80) $\boxed{\int \frac{1}{1+\sqrt[3]{1+x}}\,dx}$

$1 + x = t^3$
$dx = 3t^2\,dt$

$3\int \frac{1}{1+\sqrt[3]{t^3}}\,t^2\,dt = 3\int \frac{t^2}{t+1}\,dt$

$t^2 : t+1 = t+1 \quad \frac{-1}{t+1} \quad = 3\left(\int\left(t+1-\frac{1}{t+1}\right)dt\right)$
$\underline{-t^2 - t}$
$\quad -t$
$\quad \underline{-t - 1}$
$\qquad -1$

$= 3\left(\frac{t^2}{2} + t - \ln|t+1|\right)$

$= \boxed{3\left(\frac{(\sqrt[3]{x+1})^2}{2} + \sqrt[3]{x+1} - \ln|\sqrt[3]{x+1}+1|\right)}$

---

(81) $\boxed{\int \frac{1}{x(x+1)(x^2+3)}\,dx}$

[illegible]

$\frac{1}{x(x+1)(x^2+3)} = \frac{A}{x} + \frac{B}{x+1} + \frac{Cx+D}{x^2+3} \;\Big|\cdot x(x+1)(x^2+3)$

$1 = A(x+1)(x^2+3) + Bx(x^2+3) + (Cx+D)(x^2+x)$

$1 = A(x^3+3x+x^2+3) + B(x^3+3x) + Cx^3 + Cx^2 + Dx^2 + Dx$

$1 = x^3(A+B+C) + x^2(A+C+D) + x(3A+3B+D) + (A)$

$A + B + C = 0$
$A + C + D = 0$
$3A + 3B + D = 0$
$3A = 1$

$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 3 & 3 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 3 & 1 & 3 & 0 \\ 0 & 0 & 0 & 3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 3 & 1 & 3 & 0 \\ 0 & 0 & 0 & 3 & 1 \end{pmatrix}$

$\begin{array}{ccccc} C & B & D & A & R \end{array}$
$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 3 & 1 \end{pmatrix}$

$C + B + A = 0$
$-B + D = 0$
$4D + 3A = 0$
$3A = 1 \Rightarrow$ $\boxed{C = -\frac{1}{12},\; B = -\frac{1}{4},\; D = -\frac{1}{4},\; A = \frac{1}{3}}$

$\int \frac{\frac{1}{3}}{x}dx - \int \frac{\frac{1}{4}}{x+1}dx + \int \frac{-\frac{1}{4}x - \frac{1}{4}}{x^2+3}dx$

$\frac{1}{3}\ln x - \frac{1}{4}\ln|x+1| - \frac{1}{4}\int \frac{2x+1}{x^2+3}dx \qquad A = 2 \quad a = 1$
$\qquad B = 1 \quad b = 0$
$\qquad c = 3$

$\frac{1}{3}\ln x - \frac{1}{4}\ln|x+1| - \frac{1}{4}\left(\frac{2}{2}\ln|x^2+3| + \left(1 - \frac{0}{2}\right)\int \frac{1}{x^2+(\sqrt{3})^2}dx\right)$

$\frac{1}{3}\ln x - \frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|x^2+3| - \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\,arctg\frac{x}{\sqrt{3}} + C$

---

(82) $\boxed{\int \sqrt{x}\ln x\,dx}$

šta je s t

$x = t^2$
$dx = 2t\,dt$

$2\int \sqrt{t^2}\ln t^2\; t\,dt$

$4\int t^2\ln t\,dt \qquad u = \ln t \quad dv = t^2 dt$
$\qquad du = \frac{1}{t}dt \quad v = \int t^2 dt$
$\qquad v = \frac{t^3}{3}$

$4\cdot\left(\frac{t^3}{3}\ln t - \int \frac{1}{t}\frac{t^3}{3}dt\right)$

$4\frac{t^3}{3}\ln t - \frac{4}{3}\cdot\frac{t^3}{3}$

$\frac{4}{3}\left((\sqrt{x})^3\ln\sqrt{x} - \frac{(\sqrt{x})^3}{3}\right) = \boxed{\frac{4}{3}(\sqrt{x})^3\left(\ln\sqrt{x} - \frac{1}{3}\right) +}$

---

(83) $\boxed{\int \left(\frac{e^x+1}{e^x-1}\right)^2\frac{dx}{e^x}}$

$e^x = t$
$e^x dx = dt$
$dx = \frac{dt}{e^x} = \frac{dt}{t}$

$\int \left(\frac{t+1}{t-1}\right)^2\frac{1}{t}\,\frac{dt}{t}$

$\int \frac{t^2+2t+1}{t^2(t-1)^2}\,dt$

$\frac{t^2+2t+1}{t^2(t-1)^2} = \frac{A}{t^2} + \frac{B}{t} + \frac{C}{(t-1)^2} + \frac{D}{(t-1)} \;\Big|\cdot t^2(t-1)^2$

$t^2+2t+1 = A(t-1)^2 + Bt(t-1)^2 + Ct^2 + Dt^2(t-1)$

$t^2+2t+1 = A(t^2-2t+1) + B(t^3-2t^2+t) + Ct^2 + D(t^3-t^2)$

$t^2+2t+1 = t^3(B+D) + t^2(A-2B+C-D) + t(-2A+B) + (A)$

$B + D = 0 \qquad \boxed{D = -4}$
$A - 2B + C - D = 1 \qquad \boxed{C = 4}$
$-2A + B = 2 \qquad \boxed{B = 4}$
$A = 1 \qquad \Rightarrow \boxed{A = 1}$

$= \int \frac{1}{t^2}dt + \int \frac{4}{t}dt + \int \frac{4}{(t-1)^2}dt + \int \frac{-4}{t-1}dt$

$= -\frac{1}{t} + 4\ln t - \frac{4}{t-1} - 4\ln|t-1|$

$\boxed{= -\frac{1}{e^x} + 4\,\cancel{\ln e^x}\,4x - \frac{4}{e^x-1} - 4\ln|e^x-1| + C}$

urađena u "game over" svesci

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84) $\boxed{\int \frac{e^x+1}{e^{2x}-e^x+1}\,dx}$

$e^x = t$
$e^x dx = dt$
$dx = \frac{dt}{e^x} = \frac{dt}{t}$

$\int \frac{t+1}{t^2-t+1}\frac{dt}{t}$ $t_{1,2} = \frac{1 \pm \sqrt{1-4}}{2}$

$\frac{t+1}{t(t^2-t+1)} = \frac{A}{t} + \frac{Bt+C}{t^2-t+1} \Big| \cdot t(t^2-t+1)$

$t+1 = A(t^2-t+1) + (Bt+C)t$

$t+1 = t^2(A+B) + t(-A+C) + (A)$

$A+B=0$ $\boxed{B=-1}$
$-A+C=1$ $\boxed{C=2}$
$A = 1 \Rightarrow \boxed{A=1}$

$A=-1$ $a=1$
$B=2$ $b=-1$
$c=1$

$\int \frac{1}{t}dt + \int \frac{-t+2}{t^2-t+1}dt$

$\ln t + -\frac{1}{2}\ln|t^2-t+1| + \left(2-\frac{1}{2}\right)\int \frac{dt}{t^2-t+\frac{1}{4}-\frac{1}{4}+1}$

$\ln t - \frac{1}{2}\ln|t^2-t+1| + \frac{3}{2}\int \frac{1}{(t-\frac{1}{2})^2-(\frac{1}{2})^2+1}dt$

$\ln t - \frac{1}{2}\ln|t^2-t+1| + \frac{3}{2}\ln\frac{t-\frac{1}{2}-\frac{1}{2}}{t-\frac{1}{2}+\frac{1}{2}}$

$\ln t - \frac{1}{2}\ln|t^2-t+1| + \frac{3}{2}\ln\frac{t-1}{t}$

$\boxed{\ln e^x - \frac{1}{2}\ln|e^{2x}-e^x+1| + \frac{3}{2}\ln\frac{e^x-1}{e^x} + C}$

85) $\boxed{\int \sqrt{\frac{1-x}{x^3}}\,dx}$

~~Zadatak~~

$\int \frac{\sqrt{1-x}}{\sqrt{x^3}}dx = \int \frac{\sqrt{1-x}}{\sqrt{x}(\sqrt{x})^2}dx = \int \frac{\sqrt{1-x}}{x\sqrt{x}}dx\,\frac{\sqrt{1-x}}{\sqrt{1-x}}$

$\int \frac{1-x}{x\sqrt{x-x^2}}dx$ $x = \frac{1}{t}$
$dx = -\frac{1}{t^2}dt$

$-\int \frac{1-\frac{1}{t}}{\frac{1}{t}\sqrt{\frac{1}{t}-\frac{1}{t^2}}}\frac{1}{t^2}dt$

$-\int \frac{\frac{t-1}{t}}{\sqrt{\frac{t-1}{t^2}}}\frac{dt}{t} = -\int \frac{\frac{t-1}{t}}{\frac{\sqrt{t-1}}{\sqrt{t^2}}}\frac{dt}{t} = -\int \frac{t-1}{\sqrt{t-1}}\frac{dt}{t}$

$-\int \frac{(\sqrt{t-1})^2}{\sqrt{t-1}}dt = -\int \sqrt{t-1}\,\frac{dt}{t}$ $t-1=k^2$
$dt = 2k\,dk$

$-\int \sqrt{k^2}\cdot 2k\frac{dk}{k^2+1} = -2\int \frac{k^2}{k^2+1}dk$

$= -2\int 1 - \frac{1}{k^2+1}dk = -2(k - \operatorname{arctg} k)$

$-2\sqrt{t-1} + 2\operatorname{arctg}\sqrt{t-1}$

$\boxed{-2\sqrt{\frac{1}{x}-1} + 2\operatorname{arctg}\sqrt{\frac{1}{x}-1} + C}$

II način

$\int \frac{\sqrt{1-x}}{x\sqrt{x}}dx = \int \frac{1}{x}\sqrt{\frac{1-x}{x}}\,dx$

$\frac{1-x}{x} = t^2$ $dx = \left(\frac{1}{t^2+1}\right)' dt$
$1-x = xt^2$
$1 = xt^2 + x$ $dx = -\frac{1}{(t^2+1)^2}2t\,dt$
$1 = x(t^2+1)$
$x = \frac{1}{t^2+1}$ $dx = -\frac{2t}{(t^2+1)^2}dt$

$-2\int \frac{1}{\frac{1}{t^2+1}}\sqrt{t^2}\cdot\frac{t}{(t^2+1)^2}dt = -2\int \frac{t^2}{t^2+1}dt$

$-2\int 1 - \frac{1}{t^2+1}dt = -2(t - \operatorname{arctg} t)$

$\boxed{= -2\sqrt{\frac{1-x}{x}} + 2\operatorname{arctg}\sqrt{\frac{1-x}{x}} + C}$

86) $\boxed{\int \sin x \cdot \ln|\cos x|\,dx}$

$\cos x = t$
$-\sin x\,dx = dt$
$dx = \frac{dt}{-\sin x}$

$\int \cancel{\sin x}\ln|t|\frac{dt}{-\cancel{\sin x}}$

$-\int \ln t\,dt$ $u = \ln t$ $dv = dt$
$du = \frac{1}{t}dt$ $v = \int dt$
$v = t$

$-\left(t\ln t - \int \cancel{t}\frac{1}{\cancel{t}}dt\right)$

$-t\ln t + t = t(1-\ln t) = \boxed{\cos x(1-\ln|\cos x|)+C}$

87) $\boxed{\int \frac{x^3-2}{x(x^2+1)}\,dx}$

$x^3-2 : x^3+x = 1 - \frac{x+2}{x^3+x}$
$\underline{-x^3 \pm x}$
$-x-2$

$\int 1 - \frac{x+2}{x^3+x}dx = \boxed{x - I_1}$

$(x^3+x)y = \ldots$

$I_1 = \int \frac{x+2}{x(x^2+1)}dx$

$\frac{x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \Big| \cdot x(x^2+1)$

$x+2 = A(x^2+1) + (Bx+C)x$

$x+2 = x^2(A+B) + x(C) + (A)$

$A+B=0$ $B=-2$
$C=1$
$A=2$

$A=2$ $a=1$
$B=-1$ $b=0$
$c=1$

$\int \frac{2}{x}dx + \int \frac{-2x+1}{x^2+1}dx$

$I_1 = 2\ln x + -\frac{2}{2}\ln|x^2+1| + (1-0)\int \frac{1}{x^2+1}dx$

$I_1 = 2\ln x - \ln|x^2+1| + \operatorname{arctg} x$

$\boxed{I = x - 2\ln x + \ln|x^2+1| - \operatorname{arctg} x + C}$

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(88) $\boxed{\int \frac{\sqrt{x-1}}{\sqrt[3]{x-1}+1}\,dx}$ $\quad x-1=t^6,\ dx=6t^5dt$

$\int \frac{\sqrt{t^6}}{\sqrt[3]{t^6}+1}\,6t^5dt = 6\int \frac{t^8}{t^2+1}\,dt$

$t^8 : t^2+1 = t^6 - t^4 + t^2 - 1 + \frac{1}{t^2+1}$

$6\left(\int t^6dt - \int t^4dt + \int t^2dt - \int dt + \int \frac{1}{t^2+1}dt\right)$

$6\left(\frac{t^7}{7} - \frac{t^5}{5} + \frac{t^3}{3} - t + \operatorname{arctg} t\right)$

$\boxed{6\left(\frac{(\sqrt[6]{x-1})^7}{7} - \frac{(\sqrt[6]{x-1})^5}{5} + \frac{(\sqrt[6]{x-1})^3}{3} - \sqrt[6]{x-1} + \operatorname{arctg}\sqrt[6]{x-1} + C\right)}$

(89) $\boxed{\int \frac{(1-e^x)^2}{1-e^{2x}}\,dx}$ $\quad e^x=t,\ e^xdx=dt,\ dx=\frac{dt}{e^x}=\frac{dt}{t}$

$\int \frac{(1-t)^2}{1-t^2}\frac{dt}{t}$

$\int \frac{(1-t)^{\cancel{2}}}{t\cancel{(1-t)}(1+t)}\,dt = \int \frac{-t+1}{t^2+t}\,dt$ $\quad A=1,\ B=1 \quad a=1,\ b=1,\ c=0$

$-\frac{1}{2}\ln|t^2+t| + \left(1+\frac{1}{2}\right)\int \frac{1}{t^2+t}\,dt$

$-\frac{1}{2}\ln|t^2+t| + \frac{3}{2}\int \frac{1}{t^2+t+(\frac{1}{2})^2-(\frac{1}{2})^2}\,dt$

$-\frac{1}{2}\ln|t^2+t| + \frac{3}{2}\int \frac{1}{(t+\frac{1}{2})^2-(\frac{1}{2})^2}\,dt$

$-\frac{1}{2}\ln|t^2+t| + \frac{3}{2}\frac{1}{2\cdot\frac{1}{2}}\ln\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}$

$-\frac{1}{2}\ln|t^2+t| + \frac{3}{2}\ln\frac{t}{t+1}$

palovo?

$\boxed{-\frac{1}{2}\ln|e^{2x}+e^x| + \frac{3}{2}\ln\frac{e^x}{e^x+1} + C}$

(90) $\boxed{\int \frac{x}{x^3-8}\,dx}$ $\quad x_{1,2} = \frac{2\pm\sqrt{4-16}}{2}$

$\frac{x}{(x-2)(x^2-2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2-2x+4}$

$x = A(x^2-2x+4) + (Bx+C)(x-2)$

$x = x^2(A+B) + x(-2A-2B+C) + (4A-2C)$

$A+B=0$

$-2A-2B+C=1$

$4A-2C=0$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -2 & -2 & 1 & 1 \\ 4 & 0 & -2 & 0 \end{pmatrix}$ $/\cdot 2 / \cdot(-4)$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & -4 & -2 & 0 \end{pmatrix} \approx \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$-4B-2C=0$ $\quad \boxed{B=-1/2}$

$C=1 \Rightarrow \boxed{C=1}$

$\int \frac{\frac{1}{2}}{x-2}\,dx + \int \frac{-\frac{1}{2}x+1}{x^2-2x+4}\,dx$ $\quad A=-\frac{1}{2},\ B=1$

$= \frac{1}{2}\ln|x-2| + \left(-\frac{1}{4}\right)\ln|x^2-2x+4| + \left(1-\frac{1}{2}\right)\int \frac{1}{x^2-2x+4}$

$= \frac{1}{2}\ln|x-2| - \frac{1}{4}\ln|x^2-2x+4| + \frac{1}{2}\int \frac{1}{(x-1)^2+(\sqrt{3})^2}\,dx$

$\boxed{= \frac{1}{2}\ln|x-2| - \frac{1}{4}\ln|x^2-2x+4| + \frac{1}{2}\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x-1}{\sqrt{3}} + \ldots}$

(91) $\boxed{\int \frac{2}{(2-x)^2}\sqrt[3]{\frac{2-x}{2+x}}\,dx}$

$\frac{2-x}{2+x} = t^3$ $\quad dx = 2\cdot\frac{-3t^2(t^3+1)-(1-t^3)3t^2}{(t^3+1)^2}$

$2-x = t^3(2+x)$ $\quad dx = 2\,\frac{3t^2(-t^3-1-1+t^3)}{(t^3+1)^2}$

$2-x = 2t^3+xt^3$

$-x-xt^3 = 2t^3-2 \ /\cdot(-1)$ $\quad dx = \frac{-12t^2}{(t^3+1)^2}\,dt$

$x(1+t^3) = 2-2t^3$

$x = 2\cdot\frac{1-t^3}{t^3+1}$

$2\int \frac{1}{\left(2-2\frac{1-t^3}{t^3+1}\right)^2}\sqrt[3]{t^3}\,\frac{-12t^2}{(t^3+1)^2}\,dt$

$-\frac{24}{4}\int \frac{1}{\frac{(t^3+\cancel{1}-\cancel{1}+t^3)^2}{\cancel{(t^3+1)^2}}}\,\frac{t^3}{\cancel{(t^3+1)^2}}\,dt$

$-6\int \frac{t^3}{4t^6}\,dt = -\frac{3}{2}\int \frac{1}{t^3}\,dt$

$-\frac{3}{2}\left(\frac{-1}{2t^2}\right) = \frac{3}{4t^2} = \frac{3}{4}\left(\frac{1}{\sqrt[3]{\frac{2-x}{2+x}}}\right)^2$

$\boxed{= \frac{3}{4}\left(\sqrt[3]{\frac{2+x}{2-x}}\right)^2 + C}$

*! sea

(92) $\boxed{\int \frac{x\sqrt[3]{x+2}}{x+\sqrt[3]{x+2}}\,dx}$ $\quad x+2=t^3 \Rightarrow x=t^3-2,\ dx=3t^2dt$

$\int \frac{(t^3-2)\sqrt[3]{t^3}}{t^3-2+\sqrt[3]{t^3}}\,3t^2dt = 3\int \frac{t^6-2t^3}{t^3+t-2}\,dt$

$t^6-2t^3 : t^3+t-2 = t^3-t+\frac{t^2-2t}{t^3+t-2}$

$\frac{-t^6 \pm t^4 \mp 2t^3}{-t^4}$

$\frac{\pm t^4 \pm t^2 \mp 2t}{t^2-2t}$

$= t^3+t-2 = t^3-t+2t-2$

$= t(t^2-1)+2(t-1)$

$t(t-1)(t+1)+2(t-1)$

$(t-1)(t^2+t+2)$

$t_{1,2} = \frac{-1\pm\sqrt{1-\ldots}}{2}$

$3\int t^3dt - 3\int t\,dt + 3\int \frac{t^2-2t}{t^3+t-2}\,dt$

$\boxed{3\frac{t^4}{4} - 3\frac{t^2}{2} + 3I_1}$

ne znam sam ... *

$I_1 = \int \frac{t^2-2t}{(t-1)(t^2+t+2)}\,dt$

$\left(\sqrt[6]{x-1}\right)^3 \quad \left((x-1)^{\frac{1}{6}}\right)^3$

$$\frac{t^2-2t}{(t-1)(t^2+t+2)} = \frac{A}{t-1} + \frac{Bt+C}{t^2+t+2}$$

$$t^2-2t = A(t^2+t+2)+(Bt+C)(t-1)$$

$$t^2-2t = t^2(A+B)+t(A-B+C)+(2A-C)$$

$$\begin{aligned} A+B&=1\\ A-B+C&=-2\\ 2A-C&=0 \end{aligned} \qquad \begin{pmatrix} 1 & 1 & 0 & 1\\ 1 & -1 & 1 & -2\\ 2 & 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1\\ 0 & -2 & 1 & -3\\ 0 & -2 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 1\\ 0 & -2 & 1 & -3\\ 0 & 0 & -2 & 1 \end{pmatrix}$$

$$\begin{aligned} A+B&=1\\ -2B+C&=-3\\ -2C&=1 \end{aligned} \Rightarrow \boxed{A=-\tfrac{1}{4}} \quad \boxed{B=+\tfrac{5}{4}} \quad \boxed{C=-\tfrac{1}{2}}$$

$$\int \frac{-\frac{1}{4}}{t-1}\,dt + \int \frac{\frac{5}{4}t-\frac{1}{2}}{t^2+t+2}\,dt \qquad A=\tfrac{5}{4},\ B=-\tfrac{1}{2},\ a=1,\ b=1,\ c=2$$

$$-\frac{1}{4}\ln|t-1|+\frac{5}{8}\ln|t^2+t+2|+\left(-\frac{1}{2}-\frac{5}{8}\right)\int\frac{1}{t^2+t+2}\,dt$$

$$-\frac{1}{4}\ln|t-1|+\frac{5}{8}\ln|t^2+t+2|-\frac{9}{8}\int\frac{1}{t^2+t+(\frac{1}{2})^2-(\frac{1}{2})^2+2}\,dt$$

$$-\frac{1}{4}\ln|t-1|+\frac{5}{8}\ln|t^2+t+2|-\frac{9}{8}\int\frac{1}{(t+\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2}\,dt$$

$$-\frac{1}{4}\ln|t-1|+\frac{5}{8}\ln|t^2+t+2|-\frac{9}{8}\,\frac{1}{\frac{\sqrt{3}}{2}}\,\mathrm{arctg}\,\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$I_1 = \boxed{-\frac{1}{4}\ln|\sqrt[3]{x+2}-1|+\frac{5}{8}\ln|(\sqrt[3]{x+2})^2+\sqrt[3]{x+2}+2| - \frac{3\sqrt{3}}{4}\,\mathrm{arctg}\left(\frac{2\sqrt[3]{x+2}+1}{\sqrt{3}}\right)+C}$$

Pare 93

$$\boxed{\int \frac{\ln|\cos x|}{\sin^2 x}\,dx}$$

$u=\ln|\cos x|$, $du=\frac{1}{\cos x}(-\sin x)\,dx$, $dv=\frac{1}{\sin^2 x}$, $v=\int\frac{1}{\sin^2 x}\,dx=-\mathrm{ctg}\,x$

$$=-\mathrm{ctg}\,x\ln|\cos x| - \int -\frac{\cos x}{\sin x}\cdot\frac{-\sin x}{\cos x}\,dx$$

$$=-\mathrm{ctg}\,x\ln|\cos x| + \int dx$$

$$\boxed{=-\mathrm{ctg}\,x\ln|\cos x| + x + C}$$

Simena 94

$$\boxed{\int \frac{\ln|\mathrm{tg}\,x|}{\cos^2 x}\,dx} \qquad \mathrm{tg}\,x=t,\quad \frac{1}{\cos^2 x}dx=dt,\quad dx=\cos^2 x\,dt$$

$$=\int\frac{\ln|t|}{\cancel{\cos^2 x}}\cancel{\cos^2 x}\,dt$$

$$=\int\ln t\,dt \qquad u=\ln t,\ du=\frac{1}{t}dt,\ dv=dt,\ v=\int dt,\ v=t$$

$$=t\ln t-\int \cancel{t}\frac{1}{\cancel{t}}\,dt$$

$$=t\ln t - t = t(\ln t-1)$$

$$=\boxed{\mathrm{tg}\,x(\ln|\mathrm{tg}\,x|-1)+C}$$

$$\boxed{\int\frac{\mathrm{arctg}\,x}{x^2}\,dx} \qquad u=\mathrm{arctg}\,x,\ du=\frac{1}{x^2+1}dx,\ dv=\frac{1}{x^2}dx,\ v=\int\frac{1}{x^2}dx,\ v=-\frac{1}{x}$$

$$=-\frac{1}{x}\mathrm{arctg}\,x+\int\frac{1}{x(x^2+1)}\,dx$$

$$=\boxed{-\frac{1}{x}\mathrm{arctg}\,x+I_1}$$

$$\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}\quad /\,x(x^2+1)$$

$$1=A(x^2+1)+(Bx+C)x = x^2(A+B)+x(C)+A$$

$$A+B=0 \quad \boxed{B=-1}, \qquad C=0 \quad \boxed{C=0}, \qquad A=1 \Rightarrow \boxed{A=1}$$

$x^2+1=t$, $2x\,dx=dt$, $dx=\frac{dt}{2x}$

$$=\int\frac{1}{x}\,dx+\int\frac{-x}{x^2+1}\,dx$$

$$=\ln x-\int\frac{\cancel{x}}{t}\,\frac{dt}{2\cancel{x}}=\ln x-\frac{1}{2}\ln t$$

$$\boxed{=-\frac{1}{x}\mathrm{arctg}\,x+\ln x-\frac{1}{2}\ln|x^2+1|+C}$$

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$$\boxed{\int x\sin\sqrt{x}\,dx} \qquad x=t^2,\quad dx=2t\,dt$$

$$\int t^2\sin\sqrt{t^2}\,2t\,dt = 2\int t^3\sin t\,dt$$

$u=t^3$, $du=3t^2$, $dv=\sin t$, $v=\int\sin t$, $v=-\cos t$

$$=2\left(-t^3\cos t+\int\cos t\,3t^2\,dt\right)$$

$$=2\left(-t^3\cos t+3\int t^2\cos t\,dt\right)$$

$u=t^2$, $du=2t\,dt$, $dv=\cos t$, $v=\int\cos t$, $v=\sin t$

$$=2\left(-t^3\cos t+3\cdot\left(t^2\sin t-2\int t\sin t\,dt\right)\right)$$

$u=t$, $du=dt$, $dv=\sin t\,dt$, $v=\int\sin t\,dt$, $v=-\cos t$

$$=2\left(-t^3\cos t+3t^2\sin t-6\left(-t\cos t+\int\cos t\,dt\right)\right)$$

$$\boxed{=2\left(-(\sqrt{x})^3\cos\sqrt{x}+3x\sin\sqrt{x}+6\sqrt{x}\cos\sqrt{x}-6\sin\sqrt{x}\right)+C}$$

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$$\boxed{\int\frac{x+\sqrt{x}+\sqrt[3]{x^2}}{x(1+\sqrt[3]{x})}\,dx} \qquad x=t^6,\quad dx=6t^5\,dt$$

$$\int\frac{t^6+\sqrt{t^6}+(\sqrt[3]{t^6})^2}{t^6(1+\sqrt[3]{t^6})}\cdot 6\cancel{t^5}\,dt = 6\int\frac{t^6+t^3+t^4}{t(t^2+1)}\,dt$$

$$6\int\frac{\cancel{t}(t^5+t^3+t^2)}{\cancel{t}(t^2+1)}\,dt$$

$$(t^5+t^3+t^2):(t^2+1)=t^3+1-\frac{1}{t^2+1}$$

$$6\int\left(t^3+1-\frac{1}{t^2+1}\right)dt$$

$$6\left(\frac{t^4}{4}+t-\mathrm{arctg}\,t\right)$$

$$\boxed{6\left(\frac{(\sqrt[6]{x})^4}{4}+\sqrt[6]{x}-\mathrm{arctg}\sqrt[6]{x}\right)+C}$$

(97) $\int \frac{\dots}{\sqrt{e^{x}-1}} dx$ $\quad e^x dx = dt$, $dx = \frac{dt}{e^x} = \frac{dt}{t}$

$\int \frac{t^2}{\sqrt{t-1}} \frac{dt}{t} = \int \frac{t}{\sqrt{t-1}} dt \quad t-1=k^2,\ dt = 2k\,dk$

$2\int \frac{k^2+1}{\sqrt{k^2}} k\,dk = 2\int (k^2+1)\,dk$

$2\cdot\left(\frac{k^3}{3}+k\right) = 2\cdot\left(\frac{(\sqrt{t-1})^3}{3}+\sqrt{t-1}\right)$

$\boxed{\frac{2}{3}\left(\sqrt{e^x-1}\right)^3 + 2\sqrt{e^x-1} + C}$

---

(98) $\boxed{\int \frac{x}{x^4-16} dx}$ $\quad x^2 = t,\ 2x\,dx = dt,\ dx = \frac{dt}{2x}$

I način

$\int \frac{x}{(x^2)^2-16} dx = \int \frac{\cancel{x}}{t^2-4^2} \frac{dt}{2\cancel{x}} = \frac{1}{2}\frac{1}{2\cdot 4} \ln \frac{t-4}{t+4}$

$= \boxed{\frac{1}{16} \ln \frac{x^2-4}{x^2+4} + C}$

II način

$\frac{x}{(x^2-4)(x^2+4)} = \frac{x}{(x-2)(x+2)(x^2+4)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$

---

(99) $\boxed{\int x \ln^2 x\, dx}$ $\quad u = \ln^2 x,\ dv = x\,dx,\ du = \frac{2\ln x}{x}dx,\ v = \int x\,dx,\ v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln^2 x - 2\int \frac{x^2}{2}\frac{\ln x}{x} dx$ $\quad u = \ln x,\ dv = x\,dx,\ du = \frac{1}{x}dx,\ v = \int x\,dx,\ v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln^2 x - \frac{2}{2}\int x\ln x\, dx$

$= \frac{x^2}{2}\ln^2 x - \left(\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}dx\right)$

$= \frac{x^2}{2}\ln^2 x - \frac{x^2}{2}\ln x + \frac{1}{2}\frac{x^2}{2} + C$

$= \boxed{\frac{x^2}{2}\left(\ln^2 x - \ln x + \frac{1}{2}\right) + C}$

---

(100) $\boxed{\int e^{2x}\cos x\, dx}$ $\quad u = e^{2x},\ dv = \cos x\,dx,\ du = 2e^{2x}dx,\ v = \int \cos x\,dx,\ v = \sin x$

$I = e^{2x}\sin x - 2\int e^{2x}\sin x\, dx$ $\quad u = e^{2x},\ dv = \sin x\,dx,\ du = 2e^{2x}dx,\ v = \int \sin x\,dx,\ v = -\cos x$

šta je sa $(\cos x)'$ $(\sin x)'$

$I = e^{2x}\sin x - 2\left(-e^{2x}\cos x + 2\int e^{2x}\cos x\, dx\right)$

$I = e^{2x}\sin x + 2e^{2x}\cos x - 4I$

$4I + I = e^{2x}(\sin x + 2\cos x)$

$\boxed{I = \frac{1}{5}e^{2x}(\sin x + 2\cos x) + C}$

---

(101) $\boxed{\int \frac{x+2}{(x-2)(x^2-5x+6)} dx}$ $\quad x_{1,2} = \frac{5\pm\sqrt{25-24}}{2}$, $x_{1,2} = \begin{cases}3\\2\end{cases}$

$\int \frac{x+2}{(x-2)(x-2)(x-3)} = \int \frac{x+2}{(x-2)^2(x-3)} dx$

$\frac{x+2}{(x-2)^2(x-3)} = \frac{A}{(x-2)^2} + \frac{B}{x-2} + \frac{C}{x-3}$

$x+2 = A(x-3) + B(x-2)(x-3) + C(x-2)^2$

$x+2 = A(x-3) + B(x^2-5x+6) + C(x^2-4x+4)$

$x+2 = x^2(B+C) + x(A-5B-4C) + (-3A+6B+4C)$

$B+C=0$, $A-5B-4C=1$, $-3A+6B+4C=2$

$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & -5 & -4 & 1 \\ -3 & 6 & 4 & 2 \end{pmatrix} \approx \begin{pmatrix} 1 & -5 & -4 & 1 \\ 0 & 1 & 1 & 0 \\ -3 & 6 & 4 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & -5 & -4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -9 & -8 & 5 \end{pmatrix} /9 \approx \begin{pmatrix} 1 & -5 & -4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 \end{pmatrix}$

$A - 5B - 4C = 1$ $\quad \boxed{A = -4}$

$B + C = 0$ $\quad \boxed{B = -5}$

$C = 5$ $\quad \Rightarrow \boxed{C = 5}$

$\int \frac{-4}{(x-2)^2}dx + \int \frac{-5}{x-2}dx + \int \frac{5}{x-3}dx$

$\boxed{+\frac{4}{x-2} - 5\ln|x-2| + 5\ln|x-3| + C}$

---

(102) $\boxed{\int \frac{1}{x(x^2-2x+2)} dx}$ $\quad x_{1,2} = \frac{2\pm\sqrt{4-8}}{2}$

$\frac{1}{x(x^2-2x+2)} = \frac{A}{x} + \frac{Bx+C}{x^2-2x+2} \quad / \cdot x(x^2-2x+2)$

$1 = A(x^2-2x+2) + (Bx+C)x$

$1 = x^2(A+B) + x(-2A+C) + (2A)$

$A+B=0$ $\quad \boxed{B = -\frac{1}{2}}$

$-2A+C=0$ $\quad \boxed{C = 1}$

$2A = 1$ $\quad \Rightarrow \boxed{A = \frac{1}{2}}$

$\int \frac{\frac{1}{2}}{x}dx + \int \frac{-\frac{1}{2}x+1}{x^2-2x+2}dx$ $\quad A = -\frac{1}{2},\ B = 1;\ a = 1,\ b = -2,\ c = 2$

$\frac{1}{2}\ln|x| + \left(\frac{-\frac{1}{2}}{2\cdot 1}\right)\ln|x^2-2x+2| + \left(1-\frac{1}{2}\right)\int \frac{1}{x^2-2x+2}$

$\frac{1}{2}\ln|x| - \frac{1}{4}\ln|x^2-2x+2| + \frac{1}{2}\int \frac{1}{x^2-2x+1+1}dx$

$\frac{1}{2}\ln|x| - \frac{1}{4}\ln|x^2-2x+2| + \frac{1}{2}\int \frac{1}{(x-1)^2+1}dx$

$\boxed{\frac{1}{2}\ln|x| - \frac{1}{4}\ln|x^2-2x+2| + \frac{1}{2}\,\mathrm{arctg}\,\frac{x-1}{1} + C}$

---

(103) $\boxed{\int \ln|x^2+2x+2|\,dx}$ $\quad u = \ln(x^2+2x+2),\ dv = dx,\ du = \frac{2x+2}{x^2+2x+2}dx,\ v = x$

$= x\ln|x^2+2x+2| - \int \frac{2x^2+2x}{x^2+2x+2}dx$

$(2x^2+2x) : (x^2+2x+2) = 2 - \frac{4x}{x^2+\dots}$

$-2x^2+4x+4$

$-4x-2$

?

$= x\ln|x^2+2x+2| - \left(\int\left(2 - \frac{4x+2}{x^2+2x+2}\right)dx\right)$ $\quad A = 4,\ B = 2$

$= x\ln|x^2+2x+2| - 2x + \frac{4}{2}\ln|x^2+2x+2| + \left(2-\frac{4}{2}\right)\int \frac{1}{x^2+2x+\dots}$

$= x\ln|x^2+2x+2| - 2x + 2\ln|x^2+2x+2| - 2\int \frac{1}{(x+1)^2+1}dx$

$= \boxed{\ln|x^2+2x+2|(x+2) - 2x - 2\cdot\mathrm{arctg}\,\frac{x+1}{1} + C}$

104

$$\boxed{\int e^{3x}\sin 4x\,dx}\qquad u=e^{3x},\ du=3e^{3x},\quad dv=\sin 4x,\ v=\int\sin 4x,\ v=-\tfrac14\cos 4x$$

$$I=-\frac{e^{3x}}{4}\cos 4x+\frac34\int e^{3x}\cos 4x\,dx$$

$$u=e^{3x},\ du=3e^{3x},\quad dv=\cos 4x,\ v=\int\cos 4x\,dx,\ v=\tfrac14\sin 4x$$

$$I=-\frac{e^{3x}}{4}\cos 4x+\frac34\left(\frac{e^{3x}}{4}\sin 4x-\frac34\int e^{3x}\sin 4x\,dx\right)$$

$$I=-\frac{e^{3x}}{4}\cos 4x+\frac{3e^{3x}}{4\cdot 4}\sin 4x-\frac{9}{16}I$$

$$I+\frac{9}{16}I=\frac{e^{3x}}{4}\left(-\cos 4x+\frac34\sin 4x\right)$$

$$\frac{25}{16}I=\frac{e^{3x}}{4}\left(\frac34\sin 4x-\cos 4x\right)$$

$$\boxed{I=\frac{4}{25}e^{3x}\left(\frac34\sin 4x-\cos 4x\right)+C}$$

105

$$\boxed{\int x\cos\sqrt{x}\,dx}\qquad x=t^2,\ dx=2t\,dt$$

$$=\int t^2\cos\sqrt{t^2}\cdot 2t\,dt=2\int t^3\cos t\,dt$$

$u=t^3$, $dv=\cos t$; $du=3t^2\,dt$, $v=\sin t$

$$=2\left(t^3\sin t-3\int t^2\sin t\,dt\right)$$

$u=t^2$, $dv=\sin t\,dt$; $du=2t\,dt$, $v=-\cos t$

$$=2\left(t^3\sin t-3\cdot\left(-t^2\cos t+2\int t\cos t\,dt\right)\right)$$

$$=t^3\sin t+3t^2\cos t-6\int t\cos t\,dt$$

$u=t$, $dv=\cos t\,dt$; $du=dt$, $v=\sin t$

$$=2\left(t^3\sin t+3t^2\cos t-6\left(t\sin t-\int\sin t\,dt\right)\right)$$

$$=2\left(t^3\sin t+3t^2\cos t-6t\sin t-6\cos t\right)$$

$$\boxed{=2\left((\sqrt{x})^3\sin\sqrt{x}+3x\cos\sqrt{x}-6\sqrt{x}\sin\sqrt{x}-6\cos\sqrt{x}\right)+C}$$

ponovo Zajebant!

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106 !

$$\boxed{\int\sqrt{\frac{1-x}{x^4(x+1)}}\,dx}$$

I način

$$\int\frac{\sqrt{1-x}}{\sqrt{x^4}\cdot\sqrt{x+1}}\,dx=\int\frac{\sqrt{1-x}}{x^2\sqrt{x+1}}\,dx\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}$$

$$\int\frac{1-x}{x^2\sqrt{(1+x)(1-x)}}\,dx=\int\frac{1-x}{x^2\sqrt{1-x^2}}\,dx\qquad x=\frac1t,\ dx=-\frac{1}{t^2}dt$$

$$-\int\frac{1-\frac1t}{\frac{1}{t^2}\sqrt{1-\frac{1}{t^2}}}\frac{1}{t^2}dt=-\int\frac{\frac{t-1}{t}}{\sqrt{\frac{t^2-1}{t^2}}}dt=\int\frac{\frac{t-1}{t}}{\frac{\sqrt{t^2-1}}{\sqrt{t^2}}}dt$$

$$\int\frac{t-1}{\sqrt{t^2-1}}dt=\int\frac{t}{\sqrt{t^2-1}}dt-\int\frac{1}{\sqrt{t^2-1}}dt$$

$$\int\frac{t}{\sqrt{k^2}}\frac{k}{t}dk-\ln\left|t+\sqrt{t^2-1}\right|\qquad t^2-1=k^2,\ 2t\,dt=2k\,dk,\ dt=\frac{k}{t}dk$$

$$k-\ln\left|t+\sqrt{t^2-1}\right|$$

$$\sqrt{t^2-1}-\ln\left|t+\sqrt{t^2-1}\right|=\boxed{\sqrt{\frac{1}{x^2}-1}-\ln\left|\frac1x+\sqrt{\frac{1}{x^2}-1}\right|+C}$$

~~II način~~ ← MOŽE I OVAKO [illegible]

$$\int\sqrt{\frac{1-x}{x^4(1+x)}}\,dx=\int\frac{1}{\sqrt{x^4}}\sqrt{\frac{1-x}{1+x}}\,dx$$

$$\int\frac{1}{x^2}\sqrt{\frac{1-x}{1+x}}\,dx\qquad \frac{1-x}{1+x}=t^2$$

$1-x=xt^2+t^2$

$-x-xt^2=t^2-1\ /\cdot(-1)$

$x+xt^2=1-t^2$

$x(t^2+1)=1-t^2$

$x=\dfrac{1-t^2}{t^2+1}$

$dx=\dfrac{-2t(t^2+1)-(1-t^2)2t}{(t^2+1)^2}dt$

$dx=\dfrac{2t(-t^2-1-1+t^2)}{(t^2+1)^2}$

$dx=\dfrac{-4t}{(t^2+1)^2}dt$

$$\int\frac{1}{\left(\frac{1-t^2}{1+t^2}\right)^2}\sqrt{t^2}\,\frac{-4t}{(t^2+1)^2}dt$$

$$-4\int\frac{\cancel{(t^2+1)^2}}{(1-t^2)^2}\frac{t^2}{\cancel{(t^2+1)^2}}dt$$

$$-4\int t\cdot\frac{t}{(1-t^2)^2}dt$$

$u=t$, $dv=\dfrac{t}{(1-t^2)^2}dt$; $du=dt$, $v=\displaystyle\int\frac{t}{(1-t^2)^2}dt$ $\quad 1-t^2=k,\ -2t\,dt=dk,\ dt=\dfrac{dk}{-2t}$

$$V=\int\frac{t}{k^2}\frac{dk}{-2t}$$

$$V=-\frac12\int\frac{1}{k^2}dk$$

$$V=\frac{1}{2k}=\frac{1}{2(1-t^2)}$$

$$-4\cdot\left(\frac{t}{2(1-t^2)}-\frac12\int\frac{1}{1-t^2}dt\right)$$

$$-4\left(\frac{t}{2(1-t^2)}+\frac12\int\frac{1}{t^2-1}dt\right)$$

$$-4\left(\frac{t}{2(1-t^2)}+\frac12\cdot\frac12\ln\frac{t-1}{t+1}\right)$$

$$-\frac{2t}{(1-t^2)}-\ln\frac{t-1}{t+1}$$

$$\boxed{\frac{2\sqrt{\frac{1-x}{1+x}}}{\left(1-\frac{1-x}{1+x}\right)}-\ln\frac{\sqrt{\frac{1-x}{1+x}}-1}{\sqrt{\frac{1-x}{1+x}}+1}+C}$$

107

$$\boxed{\int\sin(\ln x)\,dx}\qquad \ln x=t\ \Rightarrow\ x=\ \ \frac1x dx=dt,\ dx=x\,dt$$

$$\int\sin t\ x\,dt$$

$$\int e^t\sin t\,dt\qquad u=e^t,\ du=e^t dt,\quad dv=\sin t\,dt,\ v=\int\sin t\,dt,\ v=-\cos t$$

$$I=-e^t\cos t+\int e^t\cos t\,dt$$

$$I=-e^t\cos t+\left(e^t\sin t-\int e^t\sin t\,dt\right)\qquad u=e^t,\ du=e^t dt,\quad dv=\cos,\ v=\int\cos,\ v=\sin$$

$$I=-e^t\cos t+e^t\sin t-I$$

$$2I=e^t(-\cos t+\sin t)$$

$$I=\frac12e^{\ln x}\left(-\cos(\ln x)+\sin(\ln x)\right)$$

$$\boxed{I=\frac{x}{2}\left(\sin(\ln x)-\cos(\ln x)\right)+C}$$

108 ✱ $\boxed{\int \frac{\ln(\cos 2x)}{2\sin^2 2x}\,dx}$ $2x = t$, $2\,dx = dt$, $dx = \frac{dt}{2}$

$\frac{1}{4}\int \frac{\ln(\cos t)}{\sin^2 t}\,dt$ $u = \ln(\cos t)$, $dv = \frac{1}{\sin^2 t}dt$

$du = \frac{1}{\cos t}(-\sin t)dt$, $V = \int \frac{1}{\sin^2 t}dt$

$\frac{1}{4}\cdot\left(-\operatorname{ctg} t\ \ln(\cos t) - \int \frac{\sin t}{\cos t}\,\frac{\cos t}{\sin t}\,dt\right)$ $V = -\operatorname{ctg} t$

$-\frac{1}{4}\operatorname{ctg} t\,\ln|\cos t| - \frac{1}{4}t$

$\boxed{-\frac{1}{4}\left(\operatorname{ctg} 2x\cdot \ln|\cos 2x| + 2x\right) + C}$

✱ 109 $\boxed{\int \frac{1}{x\sqrt{x^2+1}}\,dx}$ $x = \frac{1}{t}$, $dx = -\frac{1}{t^2}dt$

$-\int \frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}+1}}\,\frac{dt}{t^2} = -\int \frac{1}{\frac{\sqrt{1+t^2}}{\sqrt{t^2}}}\,\frac{dt}{t}$

$-\int \frac{1}{\sqrt{1+t^2}}\,dt = -\ln\left|t + \sqrt{t^2+1}\right|$

$\boxed{= -\ln\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}+1}\right| + C}$

✱ 110 $\boxed{\int \operatorname{arctg}\sqrt{x}\,dx}$ $x = t^2$, $dx = 2t\,dt$

$\int \operatorname{arctg}\sqrt{t^2}\,t\,dt$

$2\int t\operatorname{arctg} t\,dt$ $u = \operatorname{arctg} t$, $dv = t\,dt$, $du = \frac{1}{t^2+1}dt$, $V = \int t\,dt$, $V = \frac{t^2}{2}$

$2\cdot\left(\frac{t^2}{2}\operatorname{arctg} t - \int \frac{t^2}{2}\,\frac{1}{t^2+1}\,dt\right)$

$t^2\operatorname{arctg} t - \frac{2}{2}\int \frac{t^2}{t^2+1}\,dt$

$t^2\operatorname{arctg} t - \int 1 - \frac{1}{t^2+1}\,dt$

$t^2\operatorname{arctg} t - t + \operatorname{arctg} t$

$\boxed{\operatorname{arctg}\sqrt{x}\,(x+1) - \sqrt{x} + C}$

✱ 111 $\boxed{\int \sqrt{x}\,\operatorname{arctg}\sqrt{x}\,dx}$ $x = t^2$, $dx = 2t\,dt$

$2\int \sqrt{t^2}\operatorname{arctg}\sqrt{t^2}\,t\,dt$

$2\int t^2\operatorname{arctg} t\,dt$ $u = \operatorname{arctg} t$, $dv = t^2dt$, $du = \frac{1}{t^2+1}$, $V = \int t^2dt$, $V = \frac{t^3}{3}$

$2\cdot\left(\frac{t^3}{3}\operatorname{arctg} t - \int \frac{1}{3}\,\frac{t^3}{t^2+1}\,dt\right)$

$\frac{2}{3}t^3\operatorname{arctg} t - \frac{2}{3}\int 1 - \frac{t}{t^2+1}\,dt$ $t^3 : (t^2+1) = t - \frac{t}{t^2+1}$

$\frac{2}{3}t^3\operatorname{arctg} t - \frac{2}{3}t + \frac{2}{3}\int \frac{t}{t^2+1}\,dt$ $t^2+1 = k$, $2t\,dt = dk$, $dt = \frac{dk}{2t}$

$\frac{2}{3}t^3\operatorname{arctg} t - \frac{2}{3}t + \frac{2}{3}\int \frac{t}{k}\,\frac{dk}{2t}$

$\frac{2}{3}t^3\operatorname{arctg} t - \frac{2}{3}t + \frac{1}{3}\ln k$

$\frac{2}{3}t^3\operatorname{arctg} t - \frac{2}{3}t + \frac{1}{3}\ln|t^2+1|$

$\boxed{\frac{2}{3}(\sqrt{x})^3\operatorname{arctg}\sqrt{x} - \frac{2}{3}\sqrt{x} + \frac{1}{3}\ln|x+1| + C}$

108

✱ 112 $\boxed{\int \frac{\sqrt{x}}{x(1+\sqrt[3]{x})}\,dx}$ $x = t^6$, $dx = 6t^5dt$

$\int \frac{\sqrt{t^6}}{t^6(1+\sqrt[3]{t^6})}\,6t^5dt$

$6\int \frac{t^3}{t(1+t^2)}\,dt = 6\int \frac{t^2}{t^2+1}\,dt$

$= 6\int 1 - \frac{1}{t^2+1}\,dt = 6\cdot\left(t - \operatorname{arctg} t\right)$

$= \boxed{6\cdot\left(\sqrt[6]{x} - \operatorname{arctg}\sqrt[6]{x}\right) + C}$

✱ 113 $\boxed{\int \frac{\ln(\ln x)}{x}\,dx}$ $\ln x = t$, $\frac{1}{x}dx = dt$, $dx = x\,dt$

$\int \frac{\ln t}{\cancel{x}}\,\cancel{x}\,dt$

$\int \ln t\,dt$ $u = \ln t$, $dv = dt$, $du = \frac{1}{t}dt$, $V = \int dt$, $V = t$

$= t\ln t - \int \frac{1}{t}\,t\,dt$

$= t\ln t - t = t(\ln t - 1)$

$= \boxed{\ln x\,(\ln|\ln x| - 1) + C}$

✱ 114 $\boxed{\int \frac{\operatorname{arctg}(\ln x)}{x}\,dx}$ $\ln x = t$, $\frac{1}{x}dx = dt$, $dx = x\,dt$

$\int \frac{\operatorname{arctg} t}{\cancel{x}}\,\cancel{x}\,dt$

$\int \operatorname{arctg} t\,dt$ $u = \operatorname{arctg} t$, $dv = dt$, $du = \frac{1}{t^2+1}dt$, $V = \int dt$, $V = t$

$= t\operatorname{arctg} t - \int \frac{t}{t^2+1}\,dt$ $t^2+1 = k$, $2t\,dt = dk$, $dt = \frac{dk}{2t}$

$= t\operatorname{arctg} t - \int \frac{t}{k}\,\frac{dk}{2t}$

$= t\operatorname{arctg} t - \frac{1}{2}\ln k$

$= t\operatorname{arctg} t - \frac{1}{2}\ln|t^2+1|$

$= \boxed{\ln x\,\operatorname{arctg}(\ln x) - \frac{1}{2}\ln|\ln^2 x + 1| + C}$

zašto ga znaš? log

(115) $\boxed{\int x^2 \ln x \, dx}$ $u = \ln x$ $dv = x^2 dx$ $du = \frac{1}{x} dx$ $V = \int x^2 dx$ $V = \frac{x^3}{3}$

$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x} dx$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3} = \boxed{\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C}$

(116) $\boxed{\int \frac{x-3}{x^3-6x^2+10x} dx}$ $x_{1,2} = \frac{6 \pm \sqrt{36-40}}{2}$

$\frac{x-3}{x(x^2-6x+10)} = \frac{A}{x} + \frac{Bx+C}{x^2-6x+10} \quad / x(x^2-6x+10)$

$x-3 = A(x^2-6x+10) + (Bx+C)x$

$x-3 = x^2(A+B) + x(-6A+C) + (10A)$

$A+B=0$ $\boxed{B = \frac{3}{10}}$ $C = 1 - \frac{18}{10}$

$-6A+C=1$

$10A = -3 \rightarrow \boxed{A = -\frac{3}{10}}$ $\boxed{C = -\frac{8}{10}}$

$-\frac{3}{10}\int \frac{1}{x}dx + \int \frac{\frac{3}{10}x - \frac{8}{10}}{x^2-6x+10} = 0$

$-\frac{3}{10}\ln x + \frac{1}{10}\int \frac{3x-8}{x^2-6x+10}dx$ $A=3$ $B=-8$ $a=1$ $b=-6$ $c=10$

$-\frac{3}{10}\ln x + \frac{1}{10}\left(\frac{3}{2}\ln|x^2-6x+10| + \left(-8-\frac{-18}{2}\right)\int\frac{1}{x^2-6x+9-9+10}dx\right)$

$-\frac{3}{10}\ln x + \frac{1}{10}\left(\frac{3}{2}\ln|x^2-6x+10| + \int\frac{1}{(x-3)^2+1}dx\right)$

$\boxed{-\frac{3}{10}\ln x + \frac{3}{20}\ln|x^2-6x+10| + \frac{1}{10}\operatorname{arctg}(x-3) + C}$

(117) $\boxed{\int \left(\frac{x-1}{x+2}\right)^2 \frac{dx}{x}}$

$\frac{x^2-2x+1}{x(x+2)^2} = \frac{A}{x} + \frac{B}{(x+2)^2} + \frac{C}{x+2} \quad / x(x+2)^2$

$x^2-2x+1 = A(x^2+4x+4) + Bx + Cx^2+2Cx$

$x^2-2x+1 = x^2(A+C) + x(4A+B+2C) + (4A)$

$A+C=1 \Rightarrow \boxed{C = \frac{3}{4}}$?

$4A+B+2C=-2$ $B = -2 + \frac{6}{4} - 1$

$4A = 1$ $\boxed{A = \frac{1}{4}}$ $\boxed{B = -\frac{6}{4} = -\frac{3}{2}}$

NR25 KORAK

$\frac{1}{4}\int\frac{1}{x}dx - \frac{3}{4}\int\frac{1}{(x+2)^2}dx - \frac{3}{2}\int\frac{1}{x+2}dx$

$\boxed{\frac{1}{4}\ln x + \frac{3}{4}\frac{1}{x+2} - \frac{3}{2}\ln|x+2| + C}$

(118) $\boxed{\int\sqrt{4-x^2}\,dx \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}}$

$\int\frac{4-x^2}{\sqrt{4-x^2}}dx = \int\frac{4}{\sqrt{4-x^2}}dx - \int\frac{x^2}{\sqrt{4-x^2}}dx$

$= 4\int\frac{1}{\sqrt{2^2-x^2}}dx$ $I = \int x\frac{x}{\sqrt{4-x^2}}dx$

$= 4\arcsin\frac{x}{2} - \left(-x\sqrt{4-x^2} + \int\sqrt{4-x^2}\,dx\right)$

$I = 4\arcsin\frac{x}{2} + x\sqrt{4-x^2} - I$

$2I = 4\arcsin\frac{x}{2} + x\sqrt{4-x^2}$

$\boxed{I = 2\arcsin\frac{x}{2} + \frac{x}{2}\sqrt{4-x^2} + C}$

$u = x$
$du = dx$
$dv = \frac{x}{\sqrt{4-x^2}}dx$
$v = \int\frac{x}{\sqrt{4-x^2}}dx$
$4-x^2 = t^2$
$-2x\,dx = 2t\,dt$
$dx = -\frac{t}{x}dt$
$V = \int\frac{x}{\sqrt{t^2}}\cdot\frac{t}{x}dt$
$V = -\int dt = -t$
$V = -\sqrt{4-x^2}$

(119) $\boxed{\int\frac{\sin x\cdot\cos x}{\sin^4 x-\cos^4 x}dx}$ $\sin^2 x+\cos^2 x = 1$ $\sin^2 x = 1-\cos^2$

$\int\frac{\sin x\cdot\cos x}{(\sin^2 x-\cos^2 x)(\underbrace{\sin^2 x+\cos^2 x}_{1})}dx$

$\int\frac{\sin x\cos x}{\sin^2 x-\cos^2 x}dx$ 2^{-1} 2

$\int\frac{\sin x\cdot\cos x}{1-\cos^2 x-\cos^2 x}dx$

$\int\frac{\sin x\cos x}{1-2\cos^2 x}dx$ $\cos x = t$ $-\sin x\,dx = dt$ $dx = \frac{dt}{-\sin x}$

$\int\frac{\cancel{\sin x}\; t}{1-2t^2}\frac{dt}{-\cancel{\sin x}}$

$\int\frac{t}{1-2t^2}dt$ $1-2t^2 = k$ $-4t\,dt = dk$ $dt = \frac{dk}{-4t}$

$\int\frac{\cancel{t}}{k}\cdot\frac{dk}{-4\cancel{t}}$

$-\frac{1}{4}\ln k = -\frac{1}{4}\ln|1-2t^2|$

$\boxed{= -\frac{1}{4}\ln|1-2\cos^2 x| + C}$

(120) $\boxed{\int\cos^2 x\,dx}$ $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\int\frac{1}{2}(1+\cos 2x)\,dx$

$\int\frac{1}{2}dx + \int\frac{1}{2}\cos 2x\,dx$

$\frac{1}{2}x + \frac{1}{2}\frac{1}{2}\sin 2x$

$\boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$ → formulice :)

Određjeni integrali

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121. $\boxed{\int \frac{x}{1+\cos x}\,dx}$

$u = x \quad dv = \frac{1}{1+\cos x}dx$

$du = dx \quad v = \int \frac{1}{1+\cos x}dx$

$x\,tg\frac{x}{2} - \int tg\frac{x}{2}\,dx$

$x\,tg\frac{x}{2} - \int tg\,t\cdot 2dt$

$\frac{x}{2} = t$

$\frac{1}{2}dx = dt$

$dx = 2dt$

$t = tg\frac{x}{2} \quad \cos x = \frac{1-t^2}{t^2+1}$

$dx = \frac{2dt}{t^2+1}$

$V = \int \frac{1}{1+\frac{1-t^2}{t^2+1}}\cdot\frac{2dt}{t^2+1}$

$V = \int \frac{1}{\frac{t^2+1+1-t^2}{t^2+1}}\cdot\frac{2dt}{t^2+1}$

$V = \int dt = t = tg\frac{x}{2}$

$x\,tg\frac{x}{2} - 2\int \frac{\sin t}{\cos t}dt$

$\cos t = k$

$-\sin t\,dt = dk$

$dt = \frac{dk}{-\sin t}$

$x\,tg\frac{x}{2} - 2\cdot\int \frac{\cancel{\sin t}}{k}\cdot\frac{dk}{-\cancel{\sin t}}$

$x\,tg\frac{x}{2} + 2\ln k = x\,tg\frac{x}{2} + 2\ln|\cos t|$

$= \boxed{x\,tg\frac{x}{2} + 2\ln\left|\cos\frac{x}{2}\right| + C}$

---

122. $\boxed{\int \frac{x\ln|x+\sqrt{1+x^2}|}{\sqrt{1+x^2}}\,dx}$

~~[illegible]~~ parcijalna integracija

$u = \ln|x+\sqrt{x^2+1}|$

$du = \frac{1}{x+\sqrt{x^2+1}}\cdot\left(1+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right)dx$

$du = \frac{1}{x+\sqrt{x^2+1}}\cdot\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}dx$

$du = \frac{1}{\sqrt{x^2+1}}dx$

$dv = \frac{x}{\sqrt{x^2+1}}dx$

$V = \int \frac{x}{\sqrt{x^2+1}}dx \qquad x^2+1 = t^2$

$2x\,dx = 2t\,dt$

$dx = \frac{t}{x}dt$

$V = \int \frac{x}{\sqrt{t^2}}\cdot\frac{t}{x}dt$

$V = \int \frac{x}{t}\cdot\frac{t}{x}dt = \int dt = t = \sqrt{x^2+1}$

$V = \sqrt{x^2+1}$

$= \sqrt{x^2+1}\cdot\ln|x+\sqrt{x^2+1}| - \int \cancel{\sqrt{x^2+1}}\cdot\frac{1}{\cancel{\sqrt{x^2+1}}}dx$

$= \boxed{\sqrt{x^2+1}\,\ln|x+\sqrt{x^2+1}| - x + C}$

---

123. $\boxed{\int_1^2 x^2\,dx} = \frac{x^3}{3}\Big|_1^2 = \frac{2^3}{3} - \frac{1^3}{3}$

$= \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$

124. $\boxed{\int_{e^2}^{e^4}\frac{dx}{x\ln x}}$

$\ln x = t$

$\frac{1}{x}dx = dt$

$dx = x\,dt$

$\int \frac{1}{\cancel{x}t}\cancel{x}dt = \ln t = \ln|\ln x|\Big|_{e^2}^{e^4}$

$\ln|\ln e^4| - \ln|\ln e^2| = \ln 4 - \ln 2$

$= \ln\frac{4}{2} = \boxed{\ln 2}$

125. $\boxed{\int_0^{\ln 2} x e^x\,dx}$

$u = x \quad dv = e^x dx$

$du = dx \quad V = \int e^x dx$

$V = e^x$

$xe^x - \int e^x dx = xe^x - e^x = e^x(x-1)\Big|_0^{\ln 2}$

$e^{\ln 2}(\ln 2 - 1) - e^0(0-1)$

$2(\ln 2 - 1) + 1 = 2\ln 2 - 2 + 1$

$\boxed{2\ln 2 - 1}$

126. $\boxed{\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} 2xe^{x^2}dx}$

$x^2 = t$

$2x\,dx = dt$

$dx = \frac{dt}{2x}$

$\int \cancel{2x}e^t\frac{dt}{\cancel{2x}} = e^t = e^{x^2}\Big|_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}$

$e^{(\sqrt{\ln 3})^2} - e^{(\sqrt{\ln 2})^2} = e^3 - e^2 = \boxed{e^2(e-1)}$

127. $\boxed{\int_0^{\pi/2}\cos x\sin^5 x\,dx}$

$\sin x = t$

$\cos x\,dx = dt$

$dx = \frac{dt}{\cos x}$

$\int \cancel{\cos x}\,t^5\frac{dt}{\cancel{\cos x}}$

$\int t^5 dt = \frac{t^6}{6} = \frac{(\sin x)^6}{6}\Big|_0^{\pi/2}$

$\frac{(\sin\frac{\pi}{2})^6}{6} - \frac{(\sin 0)^6}{6} = \frac{1}{6} - \frac{0}{6}$

$= \boxed{\frac{1}{6}}$

$1 + \cancel{t^2} + 1\,\cancel{t^2}$